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ON THE RADIATION EFFICIENCY OF INFINITE PLATES SUBJECT TO A POINT LOAD IN WATER

J. HANNSEN SU AND R. VASUDEVAN

Signature Control Technology, Naval Surface Warfare Center, Carderock Division, 9500 *MacArthur Boulevard, West Bethesda, Maryland* 20817-5700, *U.S.A.*

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This paper describes an approach to calculate the radiation efficiency of infinite plates subject to a point load for any frequency. The results obtained by this approach are valid for the radiation efficiency of point-driven finite plates in water with real world material properties for frequencies near and above coincidence. This approach can be used to guide experimental measurements and numerical calculations of radiation efficiency as the excitation frequency is increased for finite plates in water. The radiation efficiency calculation for an infinite plate for all frequencies is derived for the first time. The limitation of thin plate theory in this application is also assessed by comparing it with the thick plate theory used in this work. An important distinct behavior above the coincidence frequency in a heavy fluid is revealed through the calculation of the radiation efficiency and explained by the dispersion relation of an infinite plate fully coupled with a fluid. In contrast to the case of light fluid (e.g., air), the radiation efficiency in a heavy fluid (e.g., water) does not approach unity in the neighbourhood of the coincidence frequency but at a significantly higher frequency.

1. INTRODUCTION

Various attempts to predict the radiation efficiency of practical panels by analytical means have been made in the past two to three decades by such investigators as Cremer, Heckl and Ungar [1] and Fahy [2] with emphasis on the radiation in air, and Blake [3] with emphasis on water. During the course of these investigations, experimental techniques for measuring the radiation efficiency have been developed and documented [1–4]. The measurement of radiation in air has normally been conducted beyond the coincidence frequency without any difficulty, since the coincidence frequency in the light fluid (around 1 kHz) is easily achieved for a typical panel with any laboratory set-up. However, it becomes a significant effort to measure the radiation efficiency in water beyond the coincidence frequency. The coincidence frequency for a half-rich metal panel, for example, is higher than 20 kHz. To simultaneously acquire data for multiple channels up to this frequency and higher becomes an issue for measurement hardware.

No results have been reported in the literature for the radiation efficiency of finite panels in water when it approaches unity. Therefore, the question remains of whether the radiation efficiency of panels in water would approach unity at coincidence frequency as it is the case in air. As a matter of fact, it has been assumed that the radiation efficiency should approach unity in both cases due to the fact that supersonic waves occur in the neighborhood of the coincidence frequency in either case regardless of the loading conditions. The radiation efficiency for practical panels in water at such high frequency may not be of practical interest. However, knowing how the radiation efficiency curve

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should behave as the excitation frequency is increased is helpful in judging the adequacy of the experimental measurement and numerical prediction. Another question is that if not, then how much higher than the coincidence frequency should the frequency be for the radiation efficiency to approach unity in the heavy fluid medium?

To answer these questions by measurement is a formidable, if not an impossible task. Furthermore, it requires additional efforts to understand or interpret the data once the measurement is complete, which is not always successful. Consequently, we chose a simple analytical model, i.e., a point load driven infinite plate and developed an approach to calculate the radiation efficiency based on the definition of radiation efficiency commonly employed in the laboratory measurement. The radiation efficiency for an infinite plate does not have much physical meaning in the low frequency range, but it approaches the radiation efficiency of finite plates as the excitation frequency is increased. This approach can thereby be used to guide the experimental measurement and/or the corresponding numerical calculation of radiation efficiency of finite plates in water as frequency is increased. Since we were concerned with the accuracy of the model near and above the coincidence frequency, Timoshenko–Mindlin plate theory was used and comparison of the results was made, whenever possible to those from well-known thin plate theory, for educational purposes.

The point-driven thin plate analytical model was popular in the late seventies [7–11] and the work most closely related to this one was that by Crighton [7]. In that work, the mathematical expression of the admittance or the response at the location of the point load was derived and discussed. However, the application of these works to calculate the radiation efficiency has never been attempted. Here we offer an approach using these established works, especially reference [5] wherein the summary of these works is most complete, first to derive a mathematical expression for the radiation efficiency of an infinite plate, and then to evaluate the expression numerically to answer these questions. This approach provides a means to calculate the radiation efficiency easily and quickly, demonstrating its validity as guidance for experimental measurements and/or numerical calculations of radiation efficiency of finite plates in water as frequency is increased. Since in most experimental measurements, unlike the case of an enclosed surface, a finite panel is fully immersed in an acoustic medium or both sides, the effect of the fluid on both sides of the plate needs to be considered. Finally, the results for fluid on both sides were also compared to those for fluid on one side.

2. APPROACH

2.1. POINT-DRIVEN INFINITE PLATE COUPLED WITH FLUID

The analysis of a Timoshenko–Mindlin plate (a.k.a., the Thick Plate Theory), driven by a point load can be simplified by taking advantage of the axisymmetry of the problem using cylindrical co-ordinates (see Figure 1). The equation for the thick plate theory with fluid pressure P is [5]

$$\left\{\left[D\nabla^2 + \frac{\rho_p h^3}{12}\omega^2\right]\left[\nabla^2 + \frac{\rho_p}{\kappa^2 G}\omega^2\right] - m\omega^2\right\}w(r)$$

$$= -\left[1 - \frac{D}{\kappa^2 G h}\nabla^2 - \frac{\rho_p h^2}{12\kappa^2 G}\omega^2\right][p(r,0) + F\delta(r)/2\pi r], \qquad (1)$$

where F is the magnitude of the point load,

$$\nabla^2 \equiv \left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right),$$

$m = \rho_p h$ is the mass of the plate per unit area, $D = Eh^3/[12(1 - \nu^2)]$ is the bending stiffness of the plate, G is the shear modulus, $\kappa^2 = \pi^2/12$ is the shear correction factor, E and ν are Young's modulus and the Poisson ratio, and ρ_p and h are the mass density and the thickness of the plate. The commonly used thin plate theory is a simplification of the thick plate theory, and the equation of motion for a thin plate loaded with a fluid on one side is [5]

$$D(\nabla^4 - k_f^4)w(r) = -p(r, 0) + F\delta(r)/2\pi r, \tag{2}$$

where $k_f^4 = m\omega^2/D$ and the acoustic pressure p should satisfy the Helmholtz equation

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + k^2\right)p(r, z) = 0, \tag{3}$$

where k is the acoustic wavenumber. The plate displacement $w(r)$ is coupled to the fluid pressure by the continuity condition

$$\frac{\partial p(r, 0)}{\partial z} = i\rho\omega\dot{w}(r), \tag{4}$$

where ρ is the fluid mass density.

The three equations (1), (3) and (4) can be solved by the Hankel transform and the solutions in terms of the transform parameter γ can be obtained as [5]

$$\tilde{w}(\gamma) = \frac{iF}{2\pi\omega(\tilde{Z}_a + \tilde{Z}_p)} \tag{5}$$

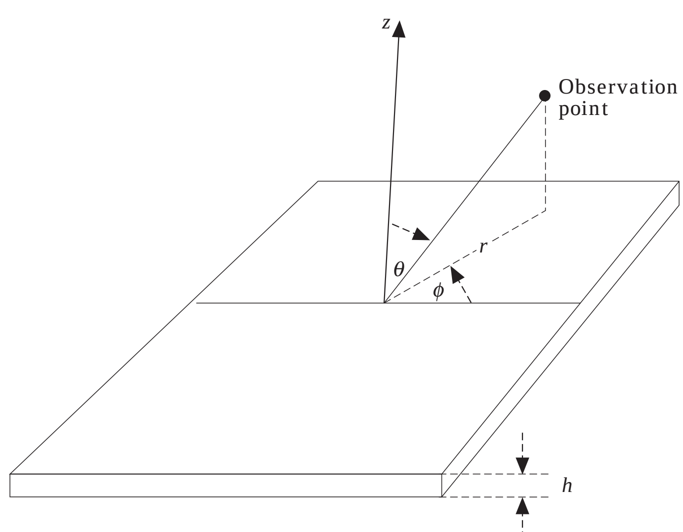

Figure 1. The co-ordinates used for the analysis and observation.

and

$$\tilde{p}(\gamma, z) = -\mathrm{i}\omega\tilde{Z}_a\,\tilde{w}(\gamma)\,\mathrm{e}^{\mathrm{i}z\sqrt{k^2-\gamma^2}}, \tag{6}$$

where

$$\tilde{Z}_a = \rho\omega/\sqrt{k^2-\gamma^2} \tag{7}$$

and

$$\tilde{Z}_p = -\mathrm{i}\omega m\,\frac{[1-(\gamma^2-k_c^2)\,(\gamma^2-k_R^2)/k_f^4]}{1+(h^2/12)\,(k_R\,/k_c)^2(\gamma^2-k_c^2)}, \tag{8}$$

where $k_c = \omega/\sqrt{E/\rho_p\,(1-\nu^2)}$ is the compressional wavenumber and $k_R = \omega/\kappa\sqrt{G/\rho_p}$ is the Rayleigh wavenumber. A simpler expression for $\tilde{Z}_p$ results if the thin plate theory is used instead:

$$\tilde{Z}_p = -\mathrm{i}\omega m(1-\gamma^4/k_f^4). \tag{9}$$

2.2. DISPERSION RELATIONS

In equations (5)–(9), the transform parameter γ's (wavenumbers) contributing to the far field result when the denominator of equation (5) vanishes:

$$\tilde{Z}_a + \tilde{Z}_p = 0, \tag{10}$$

which is the dispersion relation of an infinite plate coupled to fluid. The solution to this relation is the wavenumber γ's which can possibly exist in this coupled field. By equating the $-\tilde{Z}_p$ to $\tilde{Z}_a$ and squaring the expressions on both sides of the equation, a tenth order polynomial in terms of γ can be obtained as

$$\gamma^{10} - (k^2+2k_s^2)\gamma^8 + (k_s^4+\tau^4+2k^2k_s^2)\gamma^6 - (2\tau^4(k_s^2+k^2)+k^2k_s^4+\Re)\gamma^4 + (\tau^8+2k^2k_s^2\,\tau^4-2\aleph\Re)\gamma^2 - (k^2\tau^8+\aleph^2\Re) = 0, \tag{11}$$

where $k_s^2 = k_c^2 + k_R^2$, $k_m^4 = k_c^2\,k_R^2$, $\tau^4 = k_m^4 - k_c^4$, $\Re = [(h^2/12)\,(k_R\,/k_c)^2]\rho^2 k_f^8\,/m^2$, $\aleph = ((12/h^2k_R^2)-1)k_c^2$, and only five of the ten roots with positive real part will be considered. In addition, of the five roots, only the roots with positive imaginary part are finally selected, resulting in three types of waves in general. One represents a subsonic wave which propagates along the surface of the plate and never leaves the plate, another represents a propagating decaying wave above the coincidence frequency, which is the one most commonly referred to by acousticians, and the other represents a supersonic wave which propagates decayingly in the very low frequency range but decays fast with increased frequency and therefore is evanescent in most frequency ranges (see Figures 4 and 5).

On the other hand, with the thin plate theory, instead of solving the equation (10) directly, rewriting it in items of the variable $\xi = \sqrt{\gamma^2 - k^2}$, a fifth order polynomial of the following form can be obtained [5, 12]:

$$\xi^5 + 2k^2\xi^3 + (k^4 - k_f^4)\xi - \frac{\rho k_f^4}{m} = 0. \tag{12}$$

The solution to equation (10), γ, is then related to the roots of equation (12), ξ, through

$$\gamma = \sqrt{\xi^2 + k^2}, \tag{13}$$

where only those ξ's resulting in $\mathrm{Re}\,(\gamma) > 0$ are selected.

2.3. RADIATION EFFICIENCY CALCULATION

The radiation efficiency σ is defined as the ratio of the radiated power to the spatially averaged mean square velocity of the radiating surface. Therefore

$$\sigma = \Pi/\rho c S\hat{v}^2, \tag{14}$$

where Π is the radiated power, $\hat{v}^2$ denotes the average mean square velocity, S is the radiating surface area, and ρ and c are the density and wave speed of the acoustic medium, respectively. Due to the assumption of inviscid fluid, the radiated power can be obtained by integrating the product of the pressure and the velocity over the radiating surface or by integrating the radial component of the intensity vector over a large sphere of radius R concentric with the sound source. Here a validated expression for the radiated power derived from integrating the far-field intensity over a sphere [5] will be used,

$$\Pi = \frac{\rho R^2}{\rho c}\int_0^{\pi/2} |p(R,\theta)|^2 \sin\theta \, \mathrm{d}\theta, \tag{15}$$

where $p(R, \theta)$ can be obtained as an inverse Hankel transform of equation (6). This inverse transform can be expressed as a contour integral and evaluated in the far field by the method of stationary phase. The far-field pressure derived from this procedure for the Timoshenko–Mindlin plate theory can be shown to be [5]

$$p(R,\theta) = \frac{-\mathrm{i}kF\,\mathrm{e}^{\mathrm{i}kR}}{2\pi R} \times \frac{\cos\theta}{1 - \mathrm{i}(\omega m/\rho c)\cos\theta\left\{\dfrac{1-(k^4/k_f^4)\,[\sin^2\theta-(k_c/k)^2]\,[\sin^2\theta-(k_R/k)^2]}{1+(k^2h^2/12)\,(k_R/k_c)^2[\sin^2\theta-(k_c/k)^2]}\right\}}, \tag{16}$$

whereas for the thin plate theory [5],

$$p(R,\theta) = \frac{-\mathrm{i}kF\,\mathrm{e}^{\mathrm{i}kR}}{2\pi R}\,\frac{\cos\theta}{1-\mathrm{i}(\omega m/\rho c)\cos\theta[1-(k^4/k_f^4)\sin^4\theta]}. \tag{17}$$

Using equations (16) or (17), the radiated power of an infinite plate can be evaluated by numerically integrating equation (15) with proper choice of the numerical algorithms capable of overcoming the singularity in the denominator of the integrand.

In order to calculate the radiation efficiency of an infinite plate, the denominator of equation (14) needs to be evaluated. First, the spatially averaged mean square velocity of an infinite plate has to be defined. The spatially averaged mean square velocity for an infinite plate as a function of frequency ω can be expressed in terms of cylindrical co-ordinates as

$$\hat{v}(\omega)^2 = \lim_{r\to\infty}\frac{1}{2\pi r^2}\int_0^{2\pi}\int_0^{r} |v(r,\omega)|^2 r \, \mathrm{d}r \, \mathrm{d}\phi. \tag{18}$$

Due to axisymmetry with respect to the z-axis in this problem, the integration over ϕ is reduced to a multiplication factor of 2π. Using the surface area expression in cylindrical co-ordinates for an infinite plate, $S = \lim_{r\to\infty}\pi r^2$, the denominator of equation (14) can then be rewritten as

$$\rho c S \hat{v}^2 = \pi\rho c\int_0^{\infty} |v(r)|^2 r \, \mathrm{d}r. \tag{19}$$

To use the expression of equation (5), which is in terms of the Hankel transform parameter γ, for evaluating equation (19) in cylindrical co-ordinate r, the cylindrical co-ordinate r needs to be related to the Hankel transform parameter γ. One approach is to use the contour integral in the complex γ plane for obtaining the velocity $v(r)$ as required in equation (19). However, a simpler alternative approach based on Parseval's relation [6] for the Hankel transform,

$$\int_0^\infty |v(r)|^2 r \, \mathrm{d}r = \int_0^\infty |\tilde{v}(\gamma)|^2 \gamma \, \mathrm{d}\gamma, \tag{20}$$

was used instead. The advantage of using this form is twofold: first, the contour integral on the complex plane is avoided, second, the $\tilde{v}(\gamma)$ is simply $-\mathrm{i}\omega\tilde{w}(\gamma)$, which is a simpler integrand and is already available from equation (5). Finally, the radiation efficiency of an infinite plate when loaded with a fluid on one side can then be expressed as

$$\sigma = \Pi/\pi\rho c \int_0^\infty |\tilde{v}(\gamma)|^2 \gamma \, \mathrm{d}\gamma. \tag{21}$$

The evaluation of the integral from 0 to ∞ requires some numerical treatment and will be discussed in section 3.

When the plate is fully immersed, i.e., loaded with fluid on both sides, the acoustic pressure is simply doubled for the plate formulation [11]. This doubled acoustic pressure results in doubling the fluid density [10] in equations (7), (16) and (17). This assumption is valid even for a finite plate when it is driven at frequencies near and above coincidence since the interaction of the fluid above and below the plate, through the edges and corners, ceases to take place at these frequencies. The radiated power expression of equation (15) needs to be integrated from 0 to π to cover the top and bottom faces of the plate in addition to doubling the fluid density. Also, the radiating surface should be doubled and therefore the S in the denominator of equation (14) should be doubled, resulting in doubling the denominator of equation (21).

2.4. ADEQUACY FOR APPLICATION TO FINITE PLATES

The far-field pressure radiated by a finite plate is due to two classes of modes: the efficiently radiating modes and the resonant modes. The efficiently radiating modes are characterized by their spatial characteristics, their effective structural wavelength being larger than the acoustic wavelength at the given frequency. The pressure maxima caused by the efficiently radiating modes of a finite plate have as their envelope the pressure radiated by an infinite plate [5]. As the frequency increases, more modes become efficiently radiating and the locus of the pressure maxima becomes exactly equivalent to the far-field pressure of an infinite plate. On the other hand, resonant modes have large modal amplitudes when their resonant frequencies are close to the excitation frequency. When the excitation frequency is below the coincidence frequency of the plate, there are not resonant modes that are also efficiently radiating modes. However, when the excitation frequency is above the coincidence frequency, all the resonant modes become efficiently radiating modes, which are accounted for by the infinite plate theory.

Which class of modes is more significant for contributing to the far-field pressure when the excitation frequency is lower than the coincidence frequency? This problem was studied by Heckl [1] by neglecting the fluid loading effect (e.g., in vacuum) on the basis of a comparison of radiated power and by Junger [5] on the basis of comparing the maximum far-field pressure from these two classes of modes. Both came to the same conclusion with the following criterion: If the loss factor η of a finite plate satisfies the inequality

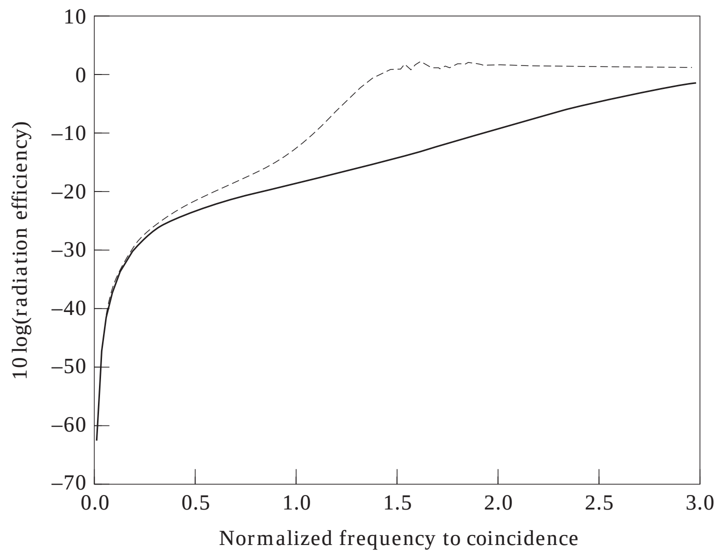

Figure 2. The calculated radiation efficiency of an infinite plate loaded with water on both sides and subjected to a point load. ——, By the "thick plate" theory; – – –, by the "thin plate" theory.

$$\eta > 8\rho\left[1+\left(\frac{\omega m}{\rho c}\right)^2\right]^{1/2}\Bigg/\left\{kk_f^2\,Am\left[1+\frac{\rho c}{\omega m}\Bigg/\sqrt{\frac{\omega_c}{\omega}-1}\right]^{3/2}\right\}, \tag{22}$$

the maximum far-field pressure from the finite plate is due to the efficiently radiating modes [5]. In equation (22), A is the area of the finite plate and ω_c is the concidence frequency and is defined as

$$\omega_c = c^2\sqrt{m/D}. \tag{23}$$

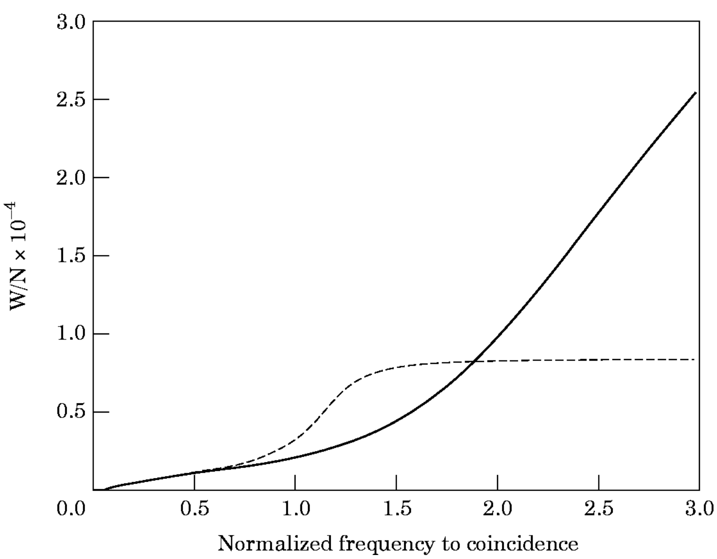

Figure 3. The calculated radiated power associated with the problem in Figure 2. ——, By the "thick plate" theory; – – –, by the "thin plate" theory.

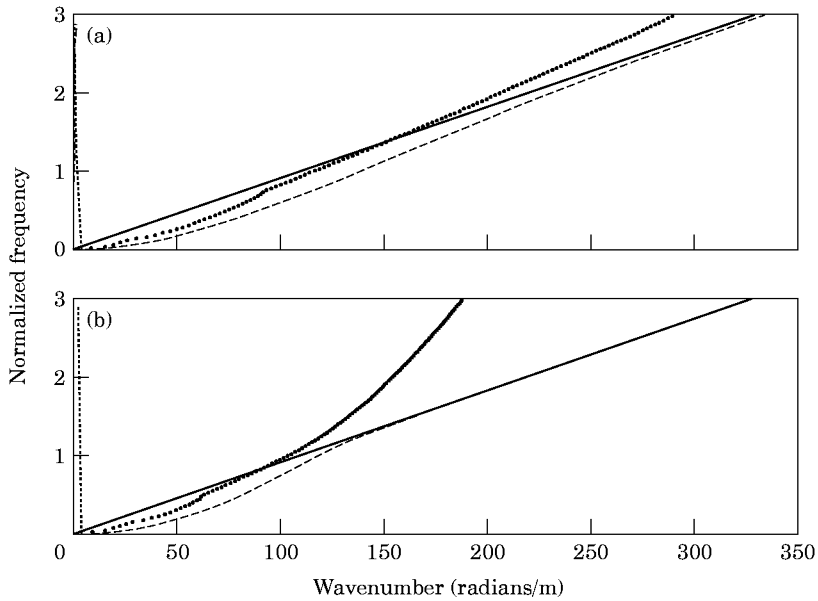

Figure 4. The calculated dispersion relation of the problem in Figure 2: (a) using the thick plate theory; (b) using the thin plate theory. ——, Acoustic wave; – – –, subsonic wave; •, supersonic wave when the frequency is above the coincidence frequency; · · ·, supersonic wave at all frequencies but with a large decaying factor.

An alternative interpretation of the criterion is that if that inequality holds, the infinite plate theory is adequate even when the excitation frequency is below the coincidence. When the excitation frequency is above the coincidence frequency, all the resonant modes becomes efficiently radiating modes and, of course, the infinite plate theory, which contains all the efficiently radiating modes, is applicable.

Using equation (22) the minimum loss factor, i.e., an equal sign instead of the greater sign, for a brass plate of 0·1524 m by 0·6096 m to approach the response of an infinite plate was calculated and is shown below in Figure 11. It has been found that this curve is not sensitive to the material properties and the physical dimensions of a finite plate. It is typical of the requirement of loss factor, as a function of frequency, for a finite plate to be approximated by an infinite plate. From Figure 11, we can conclude that for most finite plates, the assocaited radiating far-field pressure is equivalent to that of infinite plates for frequency near coincidence as long as their loss factors are higher than a very small number; e.g., 0·5%. This low factor requirement is easily met by most real world materials, and therefore the proposed approach to calculate the radiation efficiency of infinite plates can be applied to most finite plates for frequencies near and above the coincidence.

3. RESULTS AND DISCUSSION

As an example, the material properties of a brass plate of half an inch thickness (0·0127 m) are used to calculate radiation efficiency. The material properties assumed for this plate are $E = 1{\cdot}0687 \times 10^{11}$ N/m^2 with a loss factor of 0·01, a Poisson ratio of 0·37 and a mass density of 8502 kg/m^3. The coincidence frequency for this plate in water is 26 420 Hz from equation (23), based on the acoustic speed of 1524 m/s in water and the mass density of water of 1000 kg/m^3. In the case of point load disturbance in this work, all the waves that can possibly exist are excited (the "free and forced waves" [12]). This can be observed from equation (5), in which the numerator is a constant (i.e., does not contain any zero or root). Therefore, the dispersion relation in equation (10) contains all the poles contributing to the coupled system response. Dispersion relations from both the thick plate and the thin plate theories are plotted in Figure 4 below with the imaginary part (decaying factor) of the two supersonic waves shown in Figure 5 for the case of water loading on both sides.

During the course of numerical calculations, the dispersion relation also provided insight into the evaluation of the integral from 0 to ∞ in the denominator of equation (21). Since all the wavenumbers are smaller than or nearly equal to the acoustic wavenumber above the coincidence frequency, it was found to be sufficient to integrate from zero to three times the acoustic wavenumber at the coincidence frequency (i.e., from 0 to approximately 350; see Figure 4), for all frequencies of radiation efficiency calculation in this example. With this numerical treatment, the denominator of equation (21) can be numerically evaluated using the velocity $-\mathrm{i}\omega\tilde{w}(\gamma)$ from equation (5), while the numerator of equation (21) can be calculated using the radiated power expression of equation (15).

This procedure can be repeated for each frequency and for both the thick and the thin plate theories for evaluating the radiation efficiency numerically. The calculated results for both theories are shown in Figure 2 as a function of frequency. Notice that both theories predict the radiation efficiency at coincidence to be about -10 to -20 dB for an infinite brass plate (with loss factor 0·01) loaded with water on both sides. They do not approach unity until at a frequency higher than three times the coincidence frequency for the thick plate theory and around 1·5 times for the thin plate theory.

To validate these results, as a first step, let us check the radiated powers calculated from both theories; they are shown in Figure 3. Good comparison between the two theories up to 0·7 times the coincidence frequency is consistent with the study reported in reference [5]. It is interesting to note that the radiated power from the thick plate theory continues

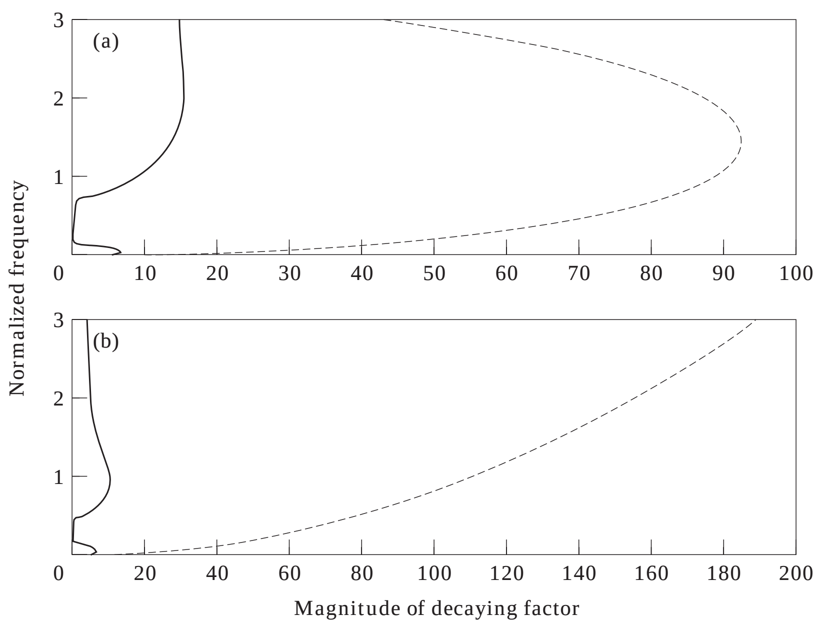

Figure 5. The imaginary part (decaying factor) of the two supersonic waves in Figure 4: (a) using the thick plate theory; (b) using the thin plate theory. ——, Supersonic wave when the frequency is above the coincidence; – – –, supersonic wave at all frequencies but with a large decaying factor.

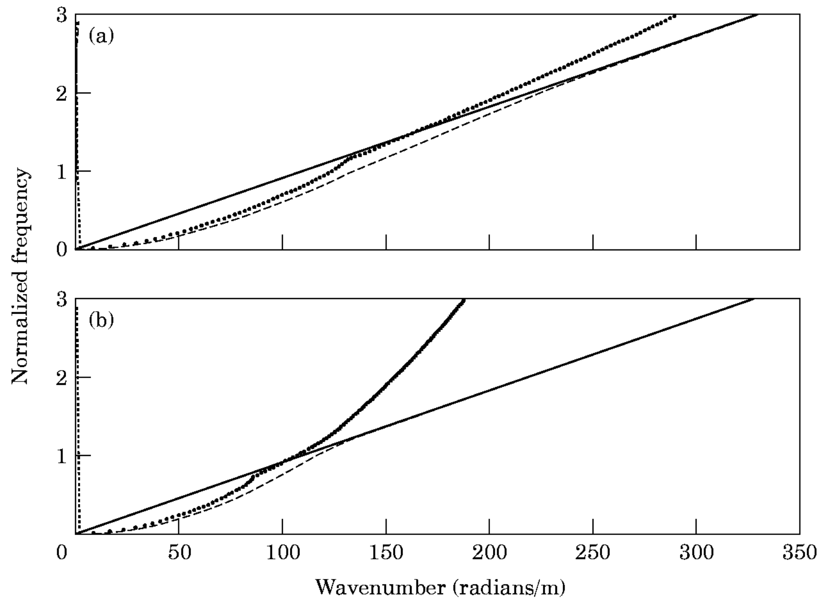

Figure 6. The calculated dispersion relation of an infinite plate loaded with water on one side and subjected to a point load: (a) using the thick plate theory; (b) using the thin plate theory. ——, Acoustic wave; – – –, subsonic wave; ●, supersonic wave when the frequency is above the coincidence frequency; · · · , supersonic wave almost at all frequencies but with a large decaying factor.

to rise with increased frequency even beyond coincidence, whereas the radiated power from the thin plate theory reaches a plateau at between 1·5 and 2·0 times the coincidence frequency. In addition, the sensitivity of the results to the material loss factor was also investigated. It was found that the radiated power is not so sensitive to the loss factor as the radiation efficiency is. A higher loss factor causes the radiation efficiency to increase more quickly with increasing frequency and therefore, the radiation efficiency would be higher at the coincidence frequency; e.g., −12 dB from the thick plate theory with a loss factor of 0·04. On the contrary, a lower loss factor reduces the radiation efficiency; e.g., −22 dB at the coincidence frequency from the thick plate theory with a loss factor of 0·005, since the magnitude of the plate velocity would be higher in the denominator of equation (21).

It would be a further validation of the results if this approach can reproduce the fact that the radiation efficiency in air approaches unity at the coincidence frequency by using the air properties for the calculation. The dispersion relation for the case of radiation in air is shown below in Figure 9, where the results from the thin plate theory are as good as the thick plate theory. This confirms the fact that thin plate theory is perfectly adequate for this frequency range (coincidence frequency in air is around 1 kHz) application. Two subsonic waves coincide with each other below the coincidence frequency; however, above the coincidence frequency, one becomes supersonic while the other becomes acoustic. The fast decaying supersonic wave is undetectable. In addition, the "supersonic wave above coincidence" almost does not decay, which is evidenced by the magnitude of the imaginary part of the two supersonic waves from the same calculation but is not shown here because of its trivial value. The ratiation efficiency for the in-air case is shown below in Figure 10 and indeed it approaches unity right after the coincidence frequency (i.e., normalized frequency 1·0). All these results reproduce the facts of radiation physics of plates in air.

Now that we are confident in the results from the developed procedure, we can then use the results to gain physical insight into the difference of the radiation efficiency between the light fluid and the heavy fluid and the limitation of the thin plate theory. First, comparing the decaying factors from both the thick plate theory and the thin plate theory in Figure 5 indicates that the much smaller decaying factors (notice the scale difference between Figures 5(a) and 5(b)) of the supersonic waves from the thick plate theory, especially right after the coincidence frequency, can yield the continuously increased radiated power as shown in Figure 3.

When the fluid loading is only on one side of the plate, the wavenumbers become a little larger below the coincidence in general but almost the same above the coincidence. On the other hand, the magnitude of decaying factors becomes smaller in general when compared to the case of water loading on both sides. These effects are demonstrated in Figures 6 and 7. As an exception worthy of note, in the very low frequency range (i.e., far below the coincidence frequency) the wavenumbers are initially smaller but increase at a faster rate as the frequency is increased, quickly exceeding the case of double-sided fluid loading. However, it is interesting to note that the decaying factor of the fast decaying supersonic wave is almost not affected. The radiated power is reduced by about 50% mainly due to the radiation from only one side of the plate; however, the radiation efficiency is not reduced by 50% since the radiating surface is now only one side of the plate. The radiation efficiency and the radiated power for one-side water loading are shown in Figure 8.

The reason why the radiation efficiency does not approach unity at the coincidence frequency in a heavy fluid (e.g., water) but at a significantly higher frequency can be explained from Figures 5 and 7. In these two figures, it can be observed that the decaying factors of the slowly decaying supersonic wave (associated with the "supersonic-above-co-

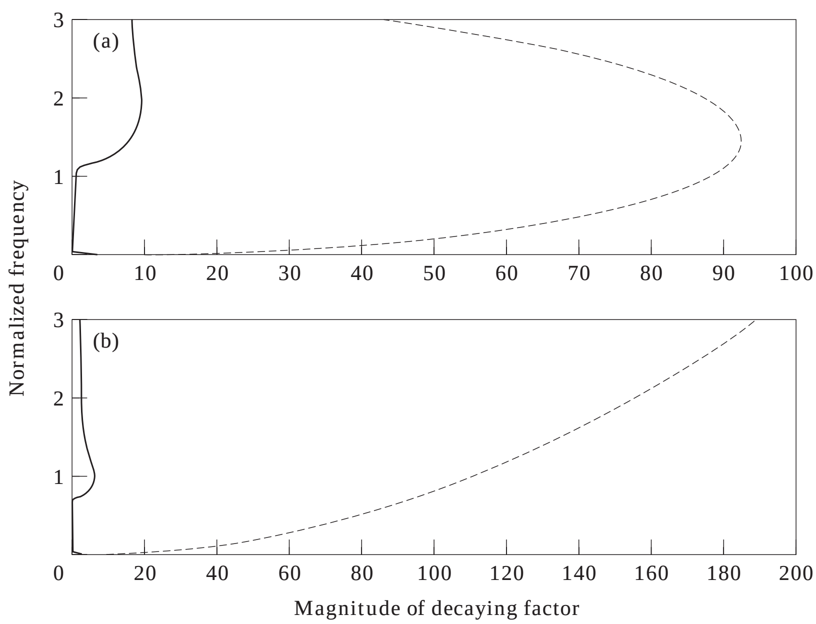

Figure 7. The imaginary part (decaying factor) of the two supersonic waves in Figure 6: (a) using the thick plate theory; (b) using the plate theory. ——, Supersonic wave when the frequency is above the coincidence; – – –, supersonic wave at all frequencies but with a large decaying factor.

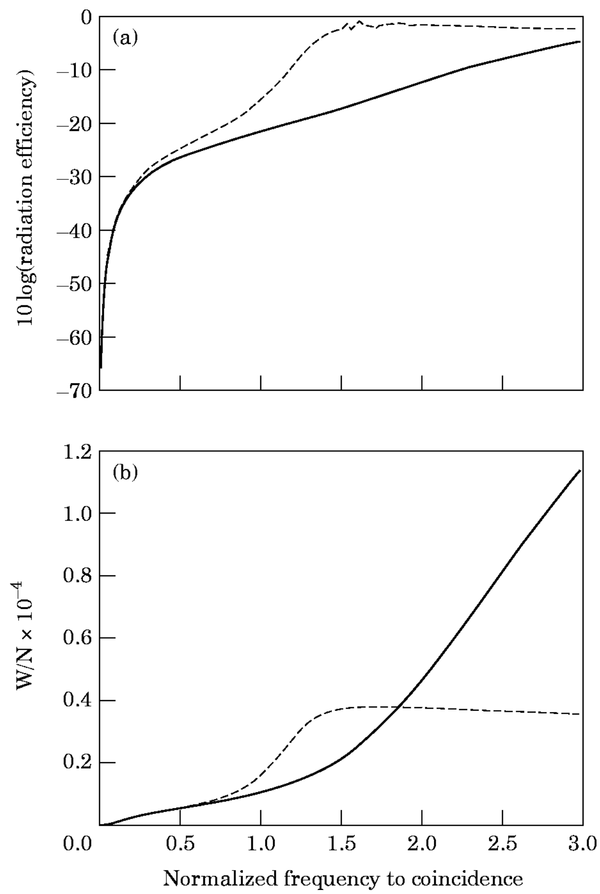

Figure 8. The calculated (a) radiation efficiency and (b) radiated power of an infinite plate loaded with water on one side and subjected to a point load. ——, By the "thick plate" theory; – – –, by the "thin plate" theory.

incidence" waves in Figures 4 and 6 respectively) are increased right after the coincidence frequency. The increase of the decaying factors implies reduction of magnitudes of the supersonic wave above the coincidence frequency. The magnitude of the supersonic wave above the coincidence frequency is responsible for the radiated power. Consequently, the reduced magnitude of the supersonic wave above the coincidence frequency causes the initial slow rising of the radiated power right after the coincidence frequency as shown in the Figures 3 and 8(b).

Furthermore, since the radiation efficiency is directly proportional to the radiated power, the radiation efficiency therefore increases slowly initially right after the coincidence frequency and ultimately delays the radiation efficiency to reach unity until at a much higher frequency. Soon after this event, the decaying factor of the supersonic wave gradually starts to decrease after the initial rise, gradually increasing the radiated power. This statement becomes evident when comparing this fact to the trivial decay of the "supersonic-above-coincidence" wave in the air loading case when there is trivial magnitude reduction (or trivial decaying factor) of the supersonic wave above the coincidence frequency. This qualitative explanation serves the purpose of gaining physical insight into this phenomenon.

Finally, the applicability of these results to a finite plate near the coincidence frequency will be illustrated by calculating the loss factor using equation (22). Again as an example, the finite brass plate with dimensions 0·1525 m by 0·6096 m with an area of 0·0929 m^2 will be used. The lower bound of the loss factor as a function of frequency for this plate in water is shown in Figure 11. Notice that the lower bound of the loss factor near the coincidence frequency (26 420 Hz) for the finite plate to behave similarly to an infinite plate is very small (smaller than 0·5%). This small loss factor can be easily met by most of the real world materials.

4. CONCLUSION

In this paper, an approach to calculate the radiation efficiency for infinite plates has been developed and validated. The comparison between the results from the thin plate theory and those from the thick plate theory indicates the thin plate theory is adequate for radiated power calculation up to 0·7 times the coincidence frequency but starts to underestimate significantly for frequencies larger than twice the coincidence frequency. As far as the radiation efficiency is concerned, the thin plate theory is adequate up to 0·2 times the coincidence frequency, and for higher frequencies, it overestimates the efficiency (see Figure 2). Consequently, it predicts a lower frequency at which the radiation efficiency approaches unity when compared to the thick plate theory. It is concluded that the heavier fluid loading causes the radiation efficiency to approach unity at a higher frequency.

The results obtained are valid for the radiation efficiency of finite plates of real material excited by a point load in water for frequencies near and above the coincidence frequency.

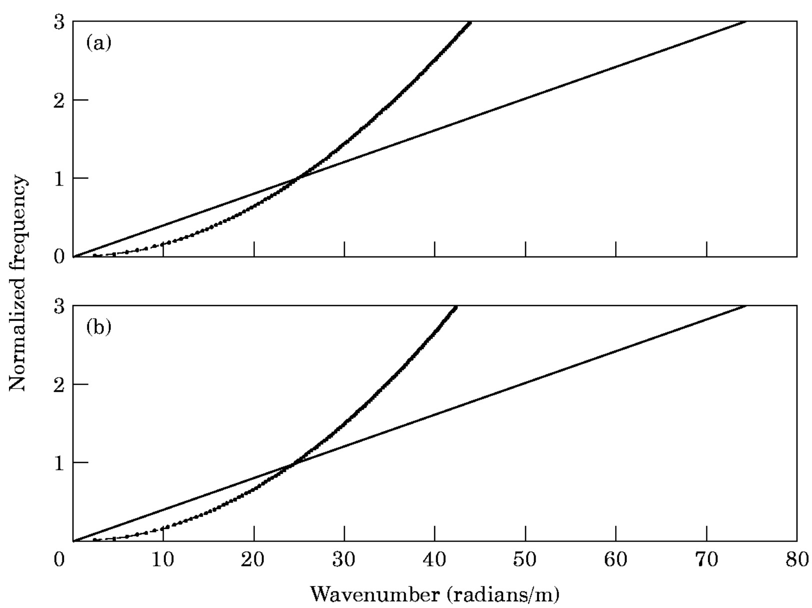

Figure 9. The calculated dispersion relation of an infinite plate loaded with air on one side and subjected to a point load: (a) using the thick plate theory; (b) using the thin plate theory. ——, Acoustic wave; – – –, subsonic wave (overlapped with the "supersonic-above-coincidence" wave below the coincidence frequency and overlapped with the acoustic wave above the coincidence frequency); ●, supersonic wave when the frequency is above the coincidence frequency.

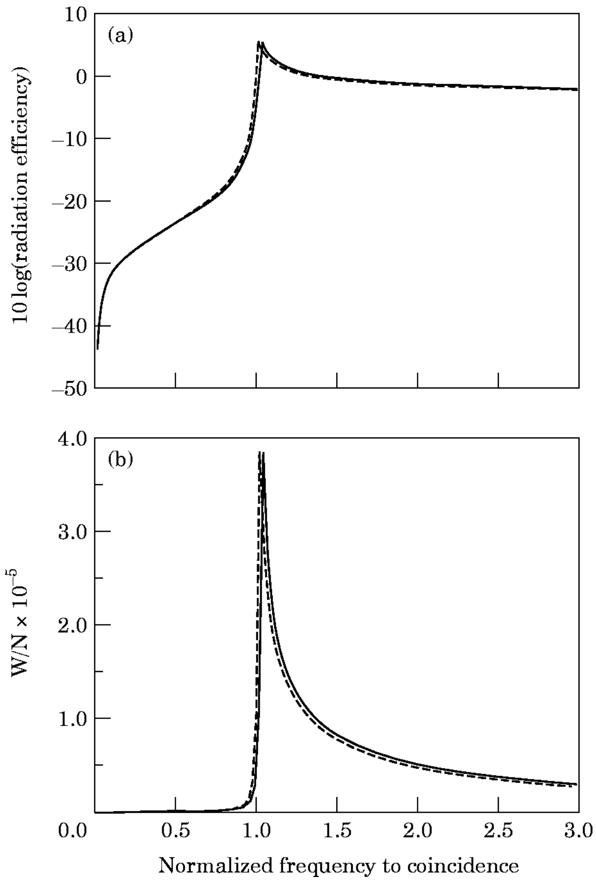

Figure 10. The calculated (a) radiation efficiency and (b) radiated power associated with the problem of Figure 9. ——, By the "thick plate" theory, – – –, by the "thin plate" theory.

This approach can thereby be used to guide the experimental measurement and/or numerical calculation of the radiation efficiency of finite plates as to what to expect of the radiation efficiency curve; e.g., how the curve should rise up and when it should approximately approach unity as a function of frequency. An important distinct behavior above the coincidence frequency in a heavy fluid is revealed by calculating the radiation efficiency with this approach and is explained by the dispersion relation of an infinite plate fully coupled with a fluid. In contrast to the case of a light fluid (e.g., air), the radiation efficiency in a heavy fluid (e.g., water) does not approach unity in the neighborhood of the coincidence frequency but at a significantly higher frequency.

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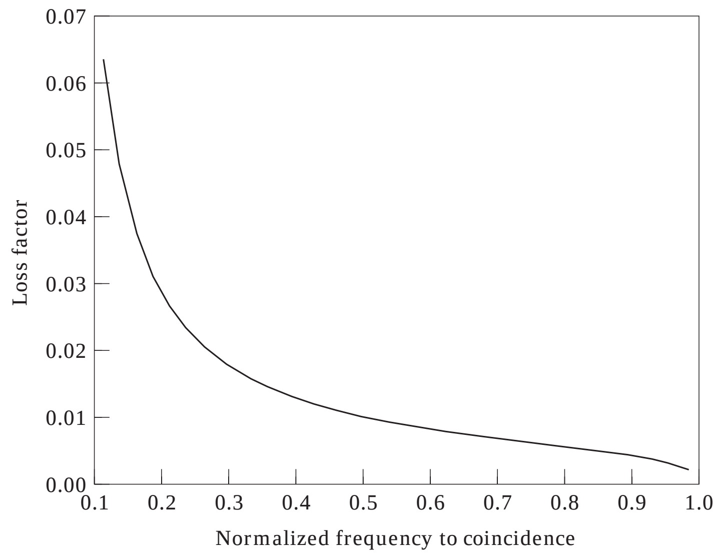

Figure 11. The required frequency dependent minimum loss factor for a finite plate to be adequately described by the infinite plate theory.

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